Cutoff Wavelengths of Waveguides with Unusual Cross Sections

This communication describes a method for computing the cut-off wavelength of waveguides of unusual cross section. This is achieved by reducing the cross section to a large number of simple rectangular strips.

For the purpose of describing the method, the writer has considered the case of a circular waveguide of radius r perturbed by two equal and diametrically opposite metal plates of penetration depth d. The electric field vector is taken as being first parallel and then perpendicular to the metal plates (see Fig. 1).

Consider first the circular waveguide without metal plates. The cross section may be reduced to N parallel steps of equal width. At the cut-off wavelength λ_c the wave impedance approaches infinity. Under this condition, it may be said that the short circuit ($Z=0$) at the boundary wall transforms to an open circuit ($Z=\infty$) at the diameter of the cross section (see Fig. 2).

Each step is in fact the cross section of a parallel plate transmission line for which the impedance relationship is

$$\frac{Z_g}{Z_1}=\frac{h_g}{h_1} \tag{1}$$

where Z_1 and h_1 are the impedance and height of the first step, respectively, Z_g and h_g are the impedance and height of the gth step, respectively, and the electrical length θ is given by

$$\theta=\frac{2\pi}{\lambda_c}\cdot\frac{r}{N}\cdot \tag{2}$$

Each length of parallel plate transmission line may be represented by the transmission matrix.[1]

$$[Z_g]=\begin{bmatrix}\cos\theta & jZ_g\sin\theta \\ \dfrac{j\sin\theta}{Z_g} & \cos\theta\end{bmatrix}. \tag{3}$$

The cross section, being composed of N parallel plate steps, may be represented by the transmission matrix.

$$[Z]=[Z_1]\times[Z_2]\times\cdots[Z_g]\times[Z_N]\cdots \tag{4}$$

Since a short circuit exists at the waveguide wall, the voltage V and current I at the diameter AB (see Fig. 2) may be represented by

$$\begin{bmatrix}V\\ I\end{bmatrix}=[Z_1]\times[Z_2]\times\cdots [Z_g]\cdots[Z_N]\times\begin{bmatrix}0\\ 1\end{bmatrix}\cdots \tag{5}$$

which reduces to

$$\begin{bmatrix}V\\ I\end{bmatrix}=\begin{bmatrix}f(\theta)\\ g(\theta)\end{bmatrix}\cdots \tag{6}$$

where $f(\theta)$ and $g(\theta)$ are polynomials in θ.

The impedance at AB approaches infinity when $g(\theta)$ is zero, *i.e.*, the solution of

$$g(\theta)=0\cdots \tag{7}$$

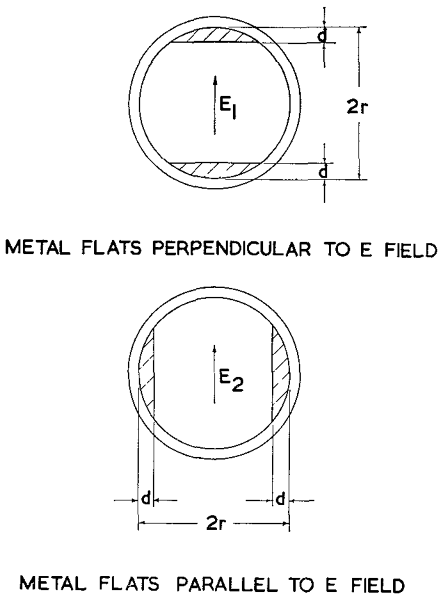

Fig. 1—Metal flats in circular waveguide.

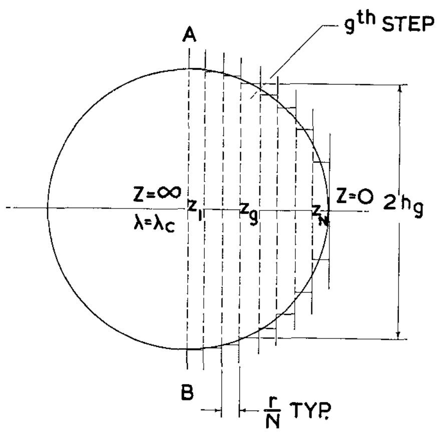

Fig. 2—Cross section of circular waveguide at cut-off wavelength λ_c.

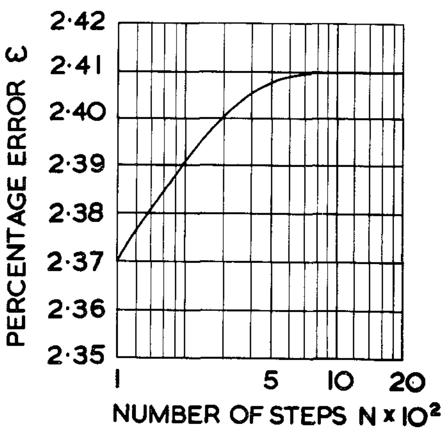

Fig. 3—Percentage error in computing λ_c as a function of the number of steps.

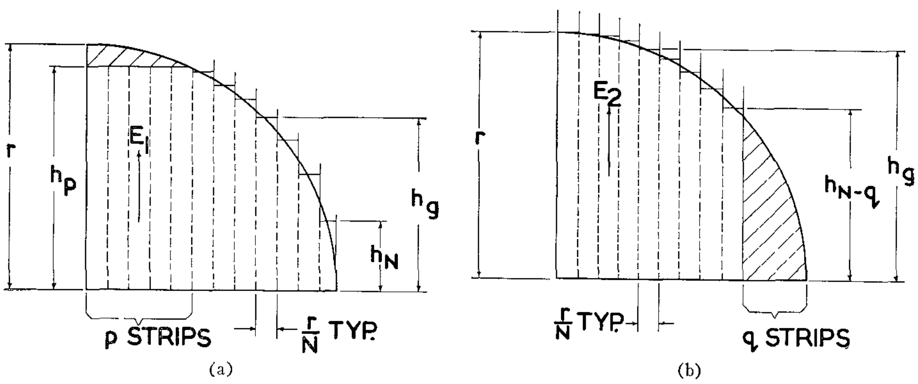

Fig. 4—Rectangular strip representation of plates parallel and perpendicular to the electric field. (a) Plates perpendicular to E field. (b) Plates parallel to E field.

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[1] R. N. Ghose, "Microwave Circuit Theory and Analysis," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed., ch. 10, 1963.

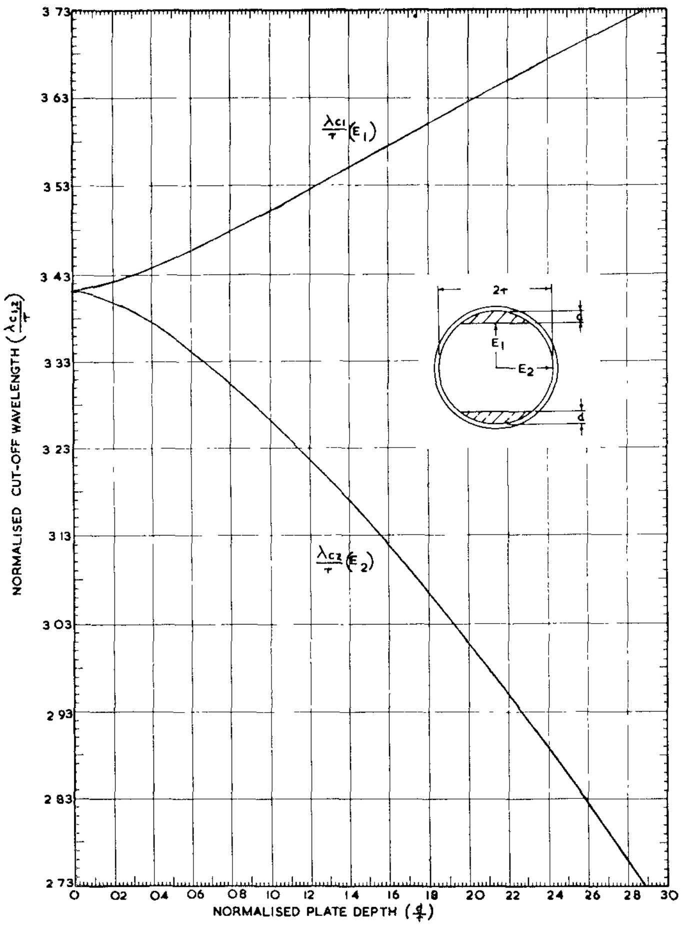

Fig. 5—Curves of cut-off wavelengths λ_{c1} and λ_{c2} as a function of plate depth, d with both coordinates normalised to radius r of circular waveguide.

gives a value of θ which, on substitution in (2), will determine the computed cut-off wavelength $\lambda_c(N)$.

In the above determination no account has been taken of the curvature of the electric field across the waveguide. Since the electric field is concentrated in the region in which it is least curved, this will lead to only a small error in determining the cut-off wavelength.

In order to determine this, error values of $\lambda_c(N)$ are computed for a range of values of N and compared with the true value of λ_c which, for the H_{11} mode, is[2]

$$\lambda_c = 2\pi r/1.841184 \cdots. \quad (8)$$

Defining the error ϵ in determining λ_c by

$$\epsilon = \frac{\lambda_c - \lambda_c(N)}{\lambda_c(N)}, \cdots \quad (9)$$

the variation of ϵ with N follows the curve shown in Fig. 3. If computations are carried out with N greater than one thousand, the value of ϵ remains steady at $2\ 41 \times 10^{-2}$, *i.e.*, there is a known bias error in the computed results.

Once the system of rectangular strips has been established, it is a simple matter to modify or remove strips in order to simulate the unusual cross section (see Fig. 4).

In the case of metal plates perpendicular to the electric field E_1, the heights of the first p strips are lowered to that of the pth strip [see Fig. 4(a)]. Proceeding as for the unperturbed circular waveguide, the cut-off wavelength λ_{c1} is determined from (7) and (2) and the predetermined bias error.

In the case of metal plates parallel to the electric field E_2, the last q strips are removed so that the short circuit appears at the end of the $(N-q)$th strip [see Fig. 4(b)]. The cut-off wavelength λ_{c2} is determined in the same way as for λ_{c1}.

The computation of λ_{c1} and λ_{c2} has been carried out on a programmed digital computer (IBM 7090). Values of N between 1400 and 2000 were used to obtain the curves shown in Fig. 5.

The method described in this communication may be applied to other waveguides of unusual cross section, provided the bias error is determined from a computation of λ_c for the closest well-known cross section.

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[2] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., ch. 8; 1943.

Circular Polarizers of Fixed Bandwidth

This communication describes the design and performance of a circular polarizer comprising two metal plates set at 45° to the electric field vector, E (Fig. 1) in a circular waveguide of radius r carrying the TE_{11} mode.

The bandwidth of this polarizer is fixed and practically independent of plate length.

At a free space wavelength λ, the differential phase shift constant, $\beta_1 - \beta_2$, may be expressed in terms of the cut-off wavelengths λ_{c1} and λ_{c2} for the component fields E_1 and E_2 (Fig. 1).

$$\beta_1 - \beta_2 = \frac{360}{\lambda}\left[\sqrt{\left(1 - \left(\frac{\lambda}{\lambda_{c1}}\right)^2\right)} - \sqrt{\left(1 - \left(\frac{\lambda}{\lambda_{c2}}\right)^2\right)}\right] \quad (1)$$

degrees per unit length.

The cut-off wavelength λ_c of the pure TE_{11} mode is[1]

$$\lambda_c = 2\pi r/1.841184 \cdots \quad (2)$$

Manuscript received May 29, 1964. This communication is published by permission of the Chief Scientist, Department of Supply, Salisbury, Australia.

[1] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., ch. 8, 1943.